Observation of the Mott Effect in Heavy Ion Collisions

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Possibility of observing the Mott momentum in the distribution of the deuterons produced in the process $p + n \rightarrow d + \gamma$, in the first stage of a nuclear reaction is presented. The correlation of a hard photon with a deuteron allows to select those deuterons produced at the beginning of a reaction.

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Recent analysis of the hard photon production in heavy ion collisions shows the importance of the two-body process $\pi + N \to N + \gamma$ in the description of the high energy part of the photon spectrum observed experimentally by the TAPS collaboration [1]. The inclusion of the processes $p+n \to d+\pi$ and $p+n \to d+\gamma$ seems to be necessary in order to improve, simultaneously, the description of the data for the pion and photon production. The two body phase-space and a weak dependence of the cross section $\sigma_{np\to d\gamma}$ on energy make the $p + N \to d + \gamma$ process important for the high energy part of the photon spectrum. The deuteron in the final state is not Pauli-blocked. If we allowed for all deuteron momenta, that process would be the dominant source of high energy photons ($E_\gamma > 30$MeV). However, the process $p+n \to d+\gamma$ requires the existence of the bound state of the final deuteron. That means, that the deuteron momentum p_d must be above the Mott momentum p_{Mott} [2–4], if the influence of the surrounding nucleons is taken into account. In particular, deuterons with low momentum cannot exist at a normal nuclear density. Typically, one finds the condition $p_d/2 > p_{Mott}/2 \simeq 1.2 p_F$ for the existence of a deuteron around the normal nuclear density [4].

The process involving a deuteron was not previously discussed in the context of hard photon production in heavy ion reactions [5]. Thus it seems important to test the relevance of that process in a more specific way, using the $d - \gamma$ correlation. The correlation to a hard photon allows us to select the deuterons from this 2-body process. Additional condition on the minimal energy of the photon or the maximal energy of deuteron allows to select the deuteron production at the early stage of a nuclear reaction. Another way of producing deuterons involves 3-body collisions [4] and it also utilizes as a basic ingredient the value of the Mott momentum. Below, we show how in a more direct way the Mott effect can be tested in heavy ion reactions using the simpler $p+n \to d+\gamma$ process, where both the photon and the deuteron in the final state can be detected experimentally.

To understand basic elements of the production process, the $d - \gamma$ correlations will be studied in a simple first-chance collision model (FCCM). In this model of the initial phase of a heavy ion reaction, the initial momentum distribution of nucleons, that limits the deuteron momenta, is given by the two Fermi spheres[1]. For that distribution, the deuterons are emitted predominantly at 90° in the nucleus-nucleus center of mass (c.m.). In describing the $p+n \to d+\gamma$ process, one needs to take into the account the interaction of the nucleon pair (the deuteron) with the surrounding medium. As shown in [2,3], the formation of bound two-particle states is strongly suppressed at large density and/or low temperature values due to Pauli blocking which limits, additionally, the deuteron momenta. To implement this effect, we parametrize the allowed region for the bound deuterons as follows [4]:

$$\int f(\frac{1}{2}\mathbf{p_d} - \mathbf{p}) \, | < \mathbf{p}|\phi > |^2 \, \frac{d^3 p}{(2\pi)^3} \ < 0.2 \quad , \tag{1}$$

where $< \mathbf{p}|\phi >$ is the deuteron wave function and $f(p)$ is the momentum distribution of the surrounding nucleons. In crude terms, the deuterons can be formed if: $p_d/2 \gtrsim p_F$, and the direction of p_d is transverse to the collision axis in the nucleus-nucleus center of mass. This is illustrated in Fig. 1; the deuteron half momentum should find itself in-between the Fermi spheres of the initial nuclei. The contributing nucleons must have large transverse momenta, in the vicinity of the Fermi momentum. The minimal transverse momentum of the emerging deuteron as a function of the energy of the collision is shown in Fig. 2. We see that around $E_{lab} = 140$MeV/A the lower limit on deuteron momentum disappears. Also at around 50MeV/A, the kinematical restriction does not allow for the production of a $d - \gamma$ pair ($E_\gamma > 30$MeV). In practice, the interesting energy range for the study of the dependence of the Mott momentum on the incident energy is $70 - 140$MeV/A. It should be noted that the value of the minimal Mott momentum depends relatively strongly on the chosen value for the Fermi momentum. In particular, if the reaction takes place at the nuclear surface, the Fermi momentum, depending on the local density, is low. The curves in Fig. 2, which correspond to different values of p_F representing the effective Fermi momentum for either a lighter or a heavier nucleus or for a low density in the nuclear surface, differ substantially. The results presented below were obtained for $p_F = 210$MeV/c.

The probability of the production of the $d - \gamma$ pair per participant can be described in the first chance collision picture by [6]:

$$\frac{dP_{\gamma d}}{d^3 p_\gamma \, d^3 p_d} = \left(\frac{1}{2} \int \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \, f(\mathbf{p}_1) \, f(\mathbf{p}_2) \, \frac{|\mathbf{p}_1 - \mathbf{p}_2|}{m} \, \frac{d\sigma_{np\to\gamma d}}{d\mathbf{p}_\gamma d\mathbf{p}_d} \, \Theta(\mathbf{p}_d, \mathbf{p}_{mott}) \right) \times$$

$$\times \left(< \sigma_{nn} >_{eff} \frac{p_{coll}}{m} \rho_0^2 \right)^{-1} \quad , \tag{2}$$

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[1]This corresponds to the $T = 0$ assumption for the nuclear medium in which the $p+n \to d+\gamma$ process takes place. In consequence, the validity of FCCM is restricted to the early stage of a reaction.

where Θ is 1 or 0 depending whether a bound deuteron state of momentum $\mathbf{p}_d$ exists or not, and where ρ_0 and $\sigma_{nn} = 40mb$ are the nuclear density and the average nucleon-nucleon cross section in medium, respectively. The first factor on the right hand side of Eq. (2) is the rate of production of $d-\gamma$ pairs per unit volume and unit time. Dividing that rate by the collision rate, we get an estimate for the number of $d-\gamma$ pairs from the first stage of a reaction per participant; $<\sigma_{NN}>_{eff}$ is the effective Pauli blocked cross section [6] and p_{coll}/m is the relative velocity of the two nuclei. Notice that, since we deal with the non-isotropic momentum distribution at the beginning of a reaction, the Mott momentum depends on the direction of the deuteron momentum, unlike in nuclear matter. Since the lowest Mott momentum is in the transverse direction (see Fig. 1), in that direction deuterons will be emitted predominantly.

For our calculations, we have parameterized the experimental data for the cross section of the deuteron breakup process, compiled in Ref. 7, with

$$\sigma_{\gamma d \to pn}[mb] = \frac{32.3}{E_\gamma^{1.39}} \quad , \quad E_\gamma < 50 MeV$$
$$= \frac{7.72}{E_\gamma^{1.01}} \quad , \quad E_\gamma > 50 MeV \ , \tag{3}$$

where E_γ is incident photon energy in MeV. The cross section for the deuteron production is then given by

$$\sigma_{pn \to \gamma d}(\sqrt{s}) = \frac{3}{2} \frac{s - 4m_N^2}{s} \sigma_{\gamma d \to pn} \ . \tag{4}$$

The momentum distribution of the produced deuteron and photon in the c.m. of the colliding $p-n$ pair is taken as isotropic.

The resulting probability distribution as a function of the relative angle between the photon and the deuteron, in the nucleus-nucleus c.m., is shown in Fig. 3. We see that the $d-\gamma$ pair is not quite emitted back to back in that frame. The angular distribution in the relative angles is broad around $180°$, with a tail extending down to very small relative angles. This is, of course, due to the fact that the nucleus-nucleus c.m. does not coincide with the c.m. of contributing nucleons; the relatively low number of back to back pairs stems from imposing the condition $E_\gamma > 30\mathrm{MeV}$ in the nucleus-nucleus c.m.[2]

The momentum distribution of the deuterons emitted in the reaction $p + n \to d + \gamma$ is shown with the solid line in the upper part of Fig. 4. Consistent with our expectations we find that the deuteron distribution is bound from below by the value of the Mott momentum and from above by the kinematical restrictions. This shape of the deuteron momentum distribution is very different from the usual spectrum of light particles emitted in nuclear reactions. The deuterons emitted in the later stage of a reaction, coming mostly from the 3-body process[3], have momenta that can extend to lower values. Also the angular distribution would be very different for all deuterons than for deuterons emitted in the first stage of a reaction. The deuterons produced in the 2-body process have momenta directed transversely in the nucleus-nucleus c.m. (Fig. 5). This reflects the existence of the gap between the two Fermi spheres which helps the formation of a bound deuteron (see Fig. 1). The 2-body process predicts a very narrow range of angles for the direction of the momentum of the deuterons observed in the correlation with a hard photon. In the laboratory frame of reference that range is around $40°$ in the forward direction for a symmetric system, i.e. for nuclei having the same effective Fermi momentum.

From the observed distribution of deuterons produced in the correlation with a hard photon, we can deduce the Mott momentum for the initial phase space configuration in a collision. The very narrow angular distribution of deuterons produced in the first stage of the collision can be used to reduce the background from deuterons produced in the 3-body collisions at latter stages of the reaction.

The anticipated background for the observation of the photon deuteron correlation involves mainly the bremsstrahlung photons and the deuterons produced in the 3-body processes. These can form background pairs either with a correlated γ or deuteron, or with themselves. The bremsstrahlung of photons is known to take place at very low rates for high energy photons [1,9], comparable to the rates of production for the $d-\gamma$ channel. This

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[2]The Mott condition (1) implies that the deuterons are mostly emitted in the direction of c.m. momentum of the colliding pair, so that the momentum of the deuteron is increased when going from the nucleus-nucleus c.m. to the nucleon-nucleon c.m. The contrary is true for most of the photons.

[3]In the latter stage of a reaction, the momentum distribution is almost isotropic. The production of a deuteron from the 2-body process requires in that case the collision of two nucleons from the tail of the momentum distribution so that total momentum of the pair is larger than $p_{Mott} \simeq 2.4 p_f$.

background can be further reduced if a supplementary condition on the photon energy is taken. The spectrum of photons in the nucleon-nucleon c.m. frame is harder than the spectrum in the nucleus-nucleus c.m. frame. Thus we propose to impose energy cuts on the energy of the photon in the $\gamma - d$ i.e. nucleon-nucleon c.m. and not on E_γ in the overall c.m., except for the 'minimal' condition: $E_\gamma > 30\text{MeV}$. Such a procedure does not reduce dramatically the cross section for the production of the the $d - \gamma$ pair (see the dashed curve in Fig. 4). On the other hand, the bremsstrahlung cross section decreases by several orders of magnitude when changing the energy of the photon in the $\gamma - d$ frame from 30 MeV to 60MeV. Thus the background coming from the bremsstrahlung photons can be neglected in comparison to the background from the deuterons produced in 3-body collisions. The deuterons produced in 3-body collisions in the first stage of the reaction have the same angular distribution as the deuterons produced by the 2-body mechanism. Accordingly, they could, with a photon coming from the $d - \gamma$ channel[4] be mistaken as a correlated pair produced in the same nucleon-nucleon collision.

The probability of the production of a deuteron from the 3-body process, per 2-body collision participant, in the impulse approximation can be written as [8,4] :

$$
\frac{dP}{d^3 p_d} = \frac{3}{4} \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 p_3}{(2\pi)^3} \, d\Omega \, f(\mathbf{p}_1) \, f(\mathbf{p}_2) \, f(\mathbf{p}_3) \, (1 - f(\mathbf{p}_1')) \frac{|\mathbf{p}_1 - \mathbf{p}_2|}{m}
$$
$$
\times \frac{d\sigma_{NN}}{d\Omega} |<\mathbf{p}_2' - \mathbf{p}_3|\phi>|^2 \, \Theta(\mathbf{p}_2' + \mathbf{p}_3, \mathbf{p}_{Mott}) \left(<\sigma_{NN}>_{eff} \frac{p_{coll}}{m} \rho_0^2 \right)^{-1} .
\tag{5}
$$

In the bottom part of Fig. 4, we show results for the probability distribution of deuterons produced in the 3-body process per 2-body collision. We see that the yields are higher than for the 2-body process, but are of the order of 10^{-7}. The probability for a statistical correlation is proportional to the product of the probabilities , thus very small, but also to the square of the number of participants, unlike the probability for true correlations which is proportional to the number of participants. Also deuterons produced at later stages of the collision would, at some rate, populate the same angular region as the deuterons produced in the first collisions, increasing the deuteron background. However, we expect that the 2-body process : $p + n \rightarrow d + \gamma$, can be extracted by the simultaneous hard photon and deuteron observation, since it leads to a very special momentum distribution of deuterons, which could be observed even if some non-negligible background is present. The deuteron distribution obtained in this manner would tell us about the Mott mechanism in nuclear matter. The photon spectrum obtained in correlation with a deuteron can be compared to the total photon yield in order to show directly which part of the spectrum is due to the photons produced in the $d - \gamma$ channel. This channel of hard photon production is important especially at energies above 140MeV [1] . Finally, it should be noted that the absorption of the deuterons is quite substantial, unlike for the absorption of photons. It can be estimated, taking $\sigma_{dN}^{ine} \simeq 2\sigma_{NN}$, that only about 20% of the produced deuterons survive in the reaction of medium size nuclei. Similar absorption rates are estimated in microscopic calculation [4]. However, the same is true also for the background deuterons, so the observation of the Mott effect in nuclear matter could be possible. We estimate that, after accounting for the absorption of deuterons, the probability of the production of a $d - \gamma$ pair is 10^{-6} per participant in the collision of medium size nuclei at energy 90MeV/A. It should be noted that the deuteron nucleon cross section in medium is not well known. One calculation [10] gave a large breakup cross section for a weakly bound deuteron, at momenta close to the Mott momentum. Another important effect may be the elastic scattering of the produced deuterons. Thus, we expect a broadening of the observed angular distributions of deuterons and of the angular spectrum of the $d - \gamma$ pairs. Finally, important could be the deuteron absorption due to the deuteron dissociation. The process may occur if the deuteron enters a higher density region, while traveling in the nuclear medium. If, in the higher density region the deuteron is not bound, it would dissociate. Similar process can occur if the gap between the two Fermi spheres (Fig. 1) becomes filled in the course of the reaction before the deuteron escapes the nuclear medium. However, the dissociation mechanism is less important than the deuteron breakup by the nucleon-deuteron collisions.

In summary, we have proposed the observation of the correlation of a hard photon with a deuteron produced in heavy ions collision. In the range of incident energies 70 MeV/A$< E_{lab} < 140$ MeV/A, the available phase space for the bound deuterons is strongly reduced by the Mott effect. This may allow for the experimental observation of the Mott momentum for deuterons. The experimentally estimated value of the Mott momentum could be used as a phenomenological parameter in the calculations of heavy ion collisions involving the formation of a deuteron in the final state. It would be also interesting to compare the observed conditions of the in medium deuteron formation to the theoretical calculations of the nuclear medium influence on the deuteron formation. The observation of the $d - \gamma$

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[4]Since, through suitable conditions on energy, the yield of bremsstrahlung photons can be made insignificant, we do not take them into account.

correlated pairs would represent a direct measurement of the photon yield produced in the reaction $p + n \rightarrow d + \gamma$, which is believed to be an important component of the total hard photon yield in intermediate energy nuclear collisions. The observation of the $d - \gamma$ correlation gives, for the first time, the possibility to estimate the percentage of the hard photon yield from a definite channel. The production of deuterons in the first stage of a reaction is determined by the allowed momenta for the deuterons (Fig. 1). The experimental observation of the correlation would tell us on the mechanism of the deuteron formation in the nuclear medium.

The main subject of this work was the implication of the Mott condition for the creation of a bound deuteron in nuclear medium on the observed spectra of deuteron. However, it should be noticed that the reaction discussed in this work: $p + n \rightarrow d + \gamma$, seems to be important also for the description of the hard photon spectra. This channel of the photon production has not been up to now addressed in the microscopic calculations of heavy ion collisions.

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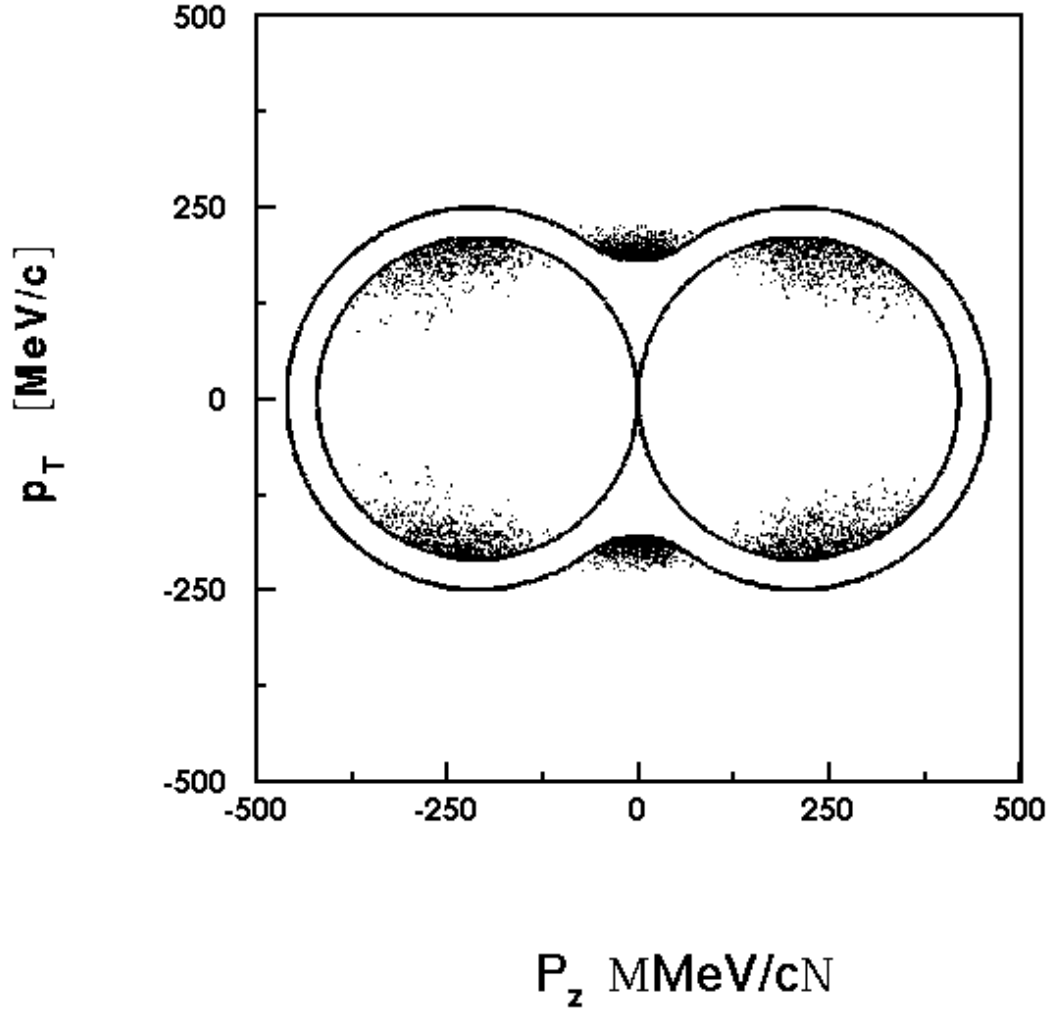[10] M. Beyer, G. Röpke and A. Sedrakian, Phys. Lett. **B376** (1996) 7.

FIG. 1. Distribution of initial longitudinal and transverse momenta of colliding nucleons (the two intersecting Fermi spheres) for $p_F = 210\text{MeV}$ at beam energy of 90MeV/A. The dots inside the spheres represent the momenta of the nucleons that have produced a deuteron in the 2-body process. A strong reduction of the cross section, in comparison to the free case, is expected due to the restriction on the phase space in the initial stage. Half of the deuteron momentum must be located outside the outer line (representing half of the angle-dependent Mott momentum). The points in the outside region represent half of the momenta of deuterons produced in correlation with a photon ($E_\gamma > 30\text{MeV}$) .
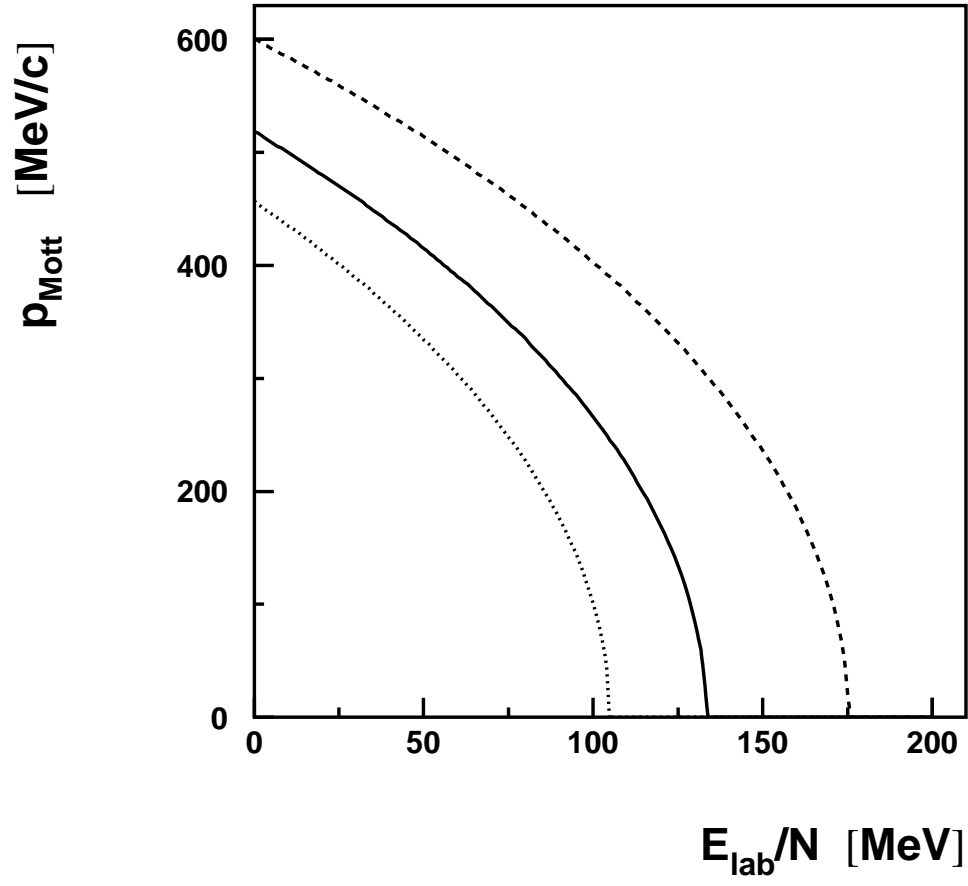
FIG. 2. The lowest value of the Mott momentum (corresponding to the transverse direction), as a function of the collision energy. The dashed, solid, and dotted lines are calculated for the Fermi momentum $p_F = 180$, 210, and 250MeV/c, respectively.
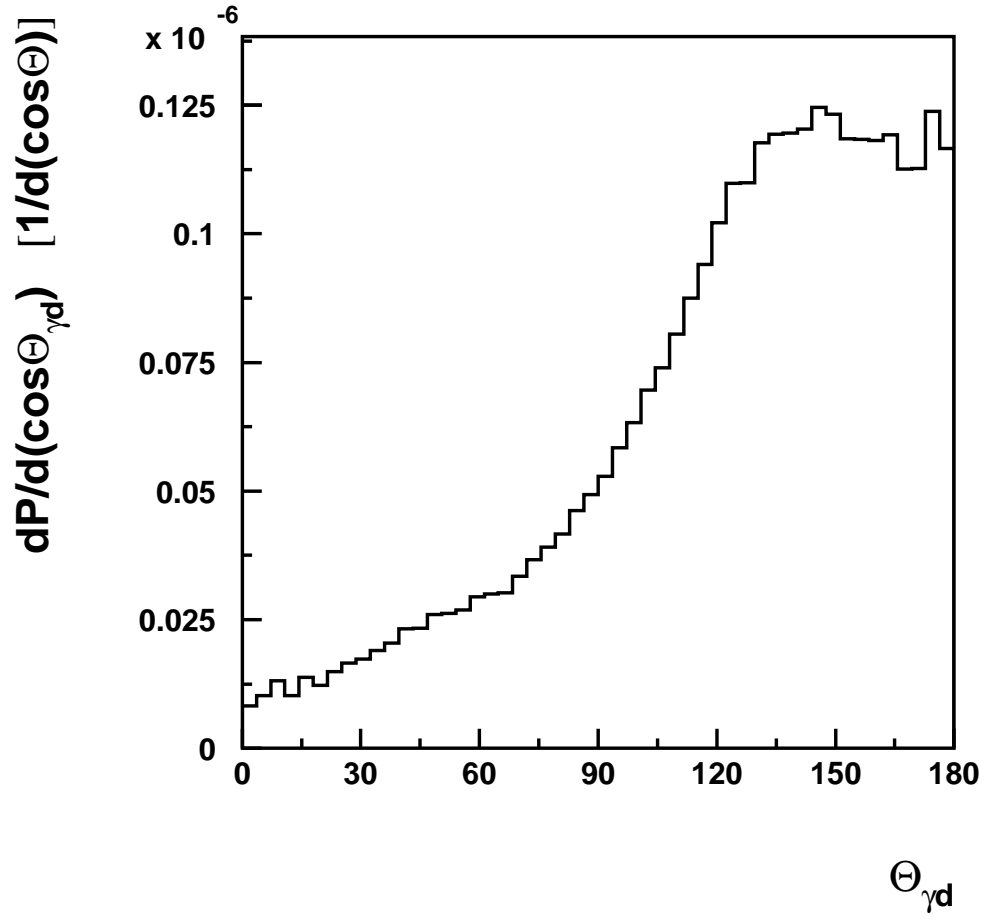
FIG. 3. Production probability of a deuteron-photon pair per participant collision as a function of the relative angle $\Theta_{\gamma d}$ at the beam energy of 90MeV/A ($E_\gamma > 30$MeV).
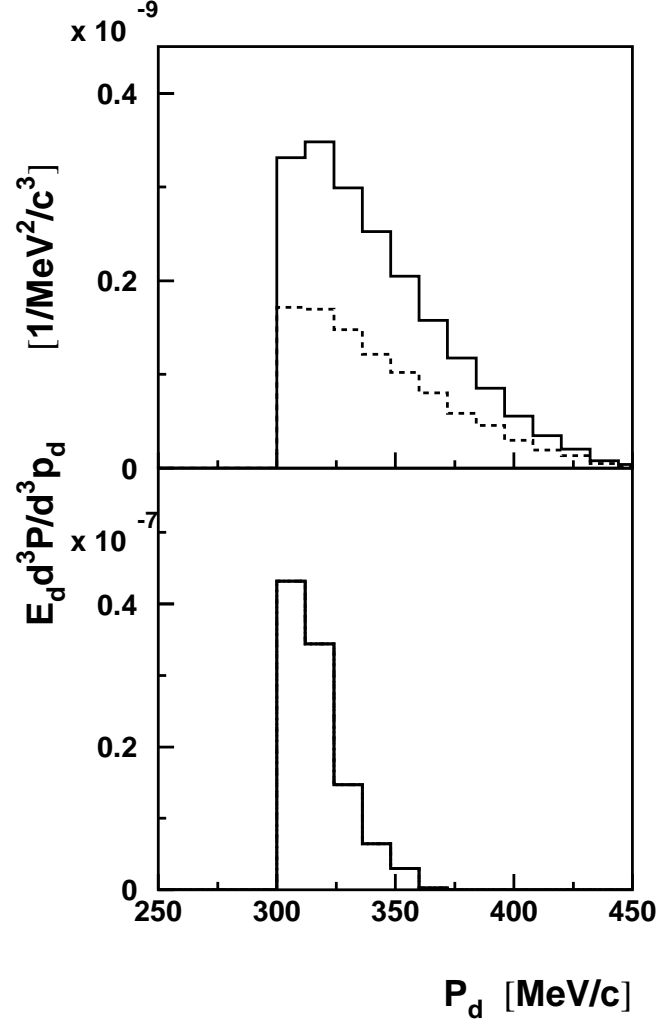
FIG. 4. Top shows the probability density for producing a deuteron in the 2-body process at 90° in the nucleon-nucleon c.m., as a function of the deuteron momentum. The solid line is obtained for the same conditions as for Fig. 3. The dashed line is obtained with the supplementary condition that the energy of the photon, in the reconstructed c.m. of the nucleon-nucleon collision, is larger than 60MeV. Bottom of the figure shows the probability density for deuteron production in the 3-body process at 90° in the nucleon-nucleon c.m., shown as a function of the deuteron momentum at the energy $90 MeV/$A.
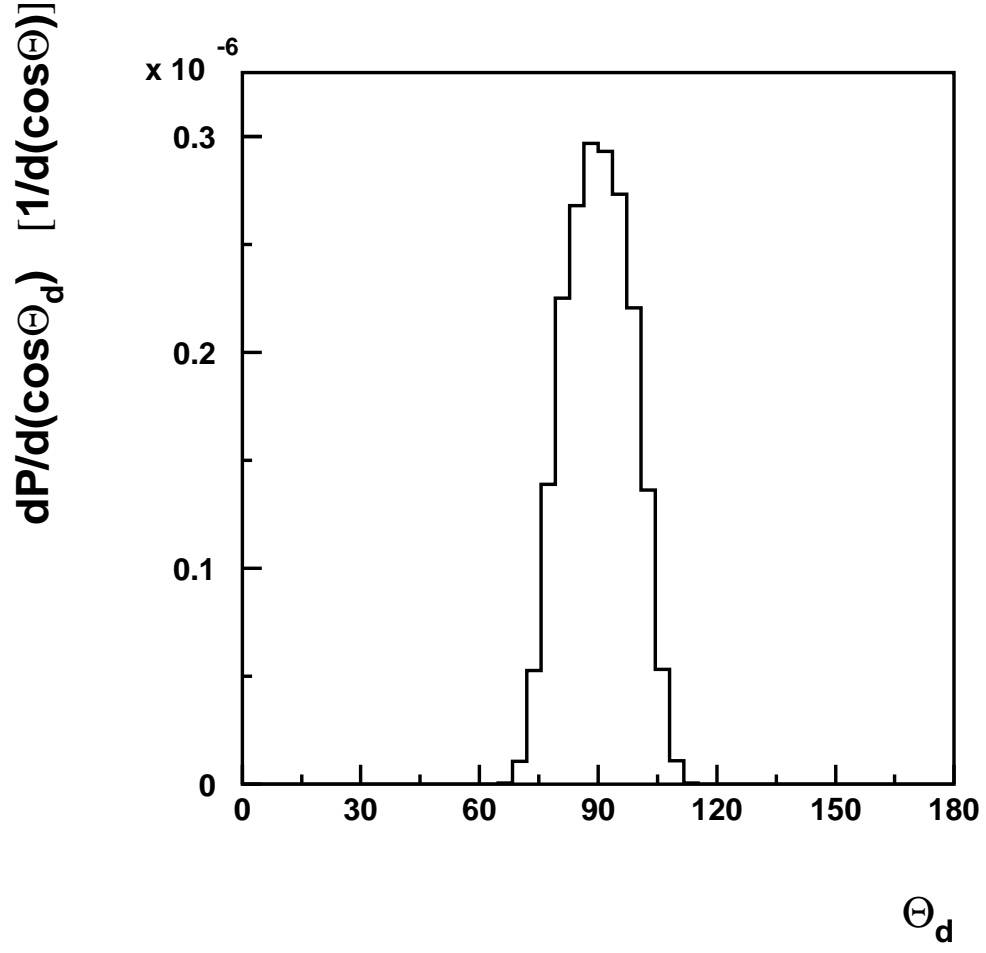
FIG. 5.   Probability density for emitted deuterons as a function of the angle in nucleus-nucleus c.m. for the same conditions as in Fig. 3.